PRINCIPLES OF ELECTROMECHANICAL ENERGY CONVERSION

1. Introduction
2. Principle of induction
3. Principle of interaction
4. Principle of alignment
5. Energy stored in magnetic field
6. Forces and torques in magnetic field systems
7. Examples (1) & (2)
8. Singly excited and multiply excited magnetic field systems
9. Inductance
10. Multiply excited magnetic field systems
11. Example (3)

References

Mulukutla S. Sarma," Electric Machines: Steady-State Theory & Dynamic Performance," West Publishing Company, St. Paul New York Los Angles San Francisco

Introduction

In this chapter we shall study the principles of electromechanical energy conversion and their application to simple devices. Electromechanical energy conversion involves the interchange of energy between an electrical and a mechanical system. When the energy is converted from electrical to mechanical form, the device is displaying *motor action*. *Generator action* involves converting mechanical energy into electrical energy. Electromechanical energy converters embody three essential features: (1) an electric system, (2) a mechanical system, and (3) a coupling field.

Both electric and magnetic fields store energy, and useful mechanical forces can be derived from them. In air or other gas at normal pressure the dielectric strength of the medium restricts the working electric field intensity to about 3×10^6 V/m, and consequently the stored electric energy density to

$$\frac{1}{2}\epsilon_0 E^2 = \frac{1}{2}\frac{10^{-9}}{36\pi}(3 \times 10^6)^2 \simeq 40 \text{ J/m}^3$$

where ϵ_0 is the permittivity of free space, given by $10^{-9}/(36\pi)$ or 8.854×10^{-12} F/m, and E is the electric field intensity. This corresponds to a force density of 40 N/m^2.

While there is no comparable restriction on magnetic fields, the saturation of ferromagnetic media required to complete the magnetic circuit limits the working magnetic flux density to about 1.6 T, for which the stored magnetic energy density in air is about

$$\frac{1}{2}\frac{B^2}{\mu_0} = \frac{1}{2}\frac{(1.6)^2}{4\pi \times 10^{-7}} \simeq 1 \times 10^6 \text{ J/m}^3$$

where μ_0 is the permeability of free space, and B is the magnetic flux density. As this is nearly 25,000 times as much as for the electric field, almost all industrial electric machines are magnetic in principle and are magnetic-field devices.

Principle of induction

Three basic principles associated with all electromagnetic devices are (1) *induction*, (2) *interaction*, and (3) *alignment*.

PRINCIPLE OF INDUCTION

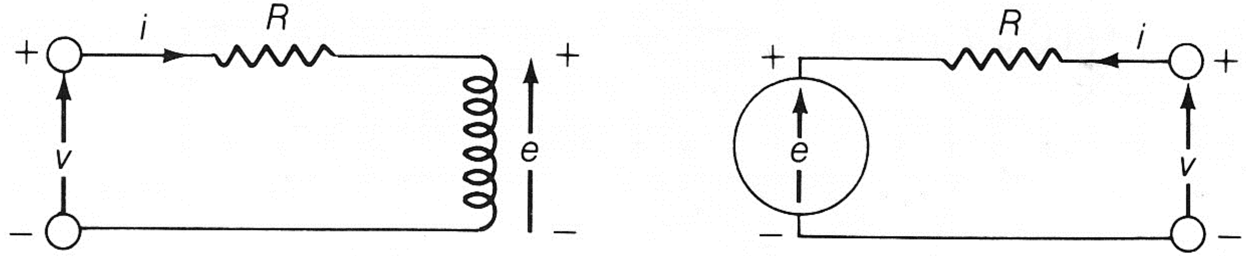
The induced emf is given by Faraday's law of induction:

$$e = -\frac{d\lambda}{dt} = -N\frac{d\phi}{dt}$$

The induced emf will be acting in the direction of positive current as shown in Figure (*b*) with a source (or generator) convention. Sometimes it is more convenient to consider the emf as directed in opposition to positive current, as shown in *(a)* with a load (or sink, or motor) convention, in which case

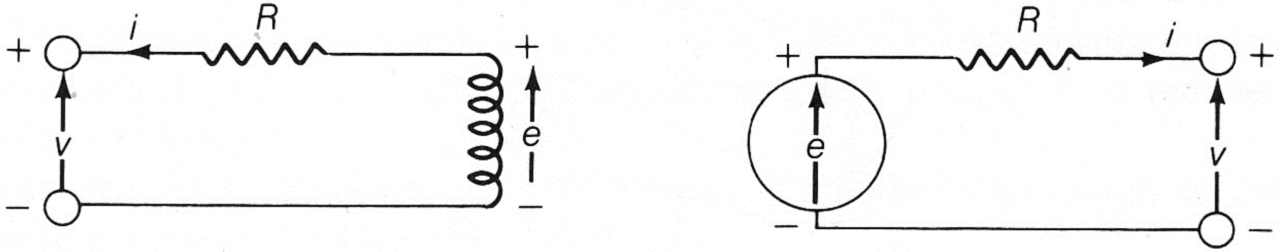
$$e = +\frac{d\lambda}{dt} = +N\frac{d\phi}{dt}$$

Circuit conventions: *(a)* load (or sink, or motor) convention (note that the power *into* the circuit is positive when v and i are positive), *(b)* source (or generator) convention (note that the power *delivered* by this circuit to the external circuit is positive when v and i are positive).



$$v = Ri + e = Ri + \frac{d\lambda}{dt} = Ri + N\frac{d\phi}{dt}$$

(a)



$$v = e - iR = -\frac{d\lambda}{dt} - iR = -N\frac{d\phi}{dt} - iR$$

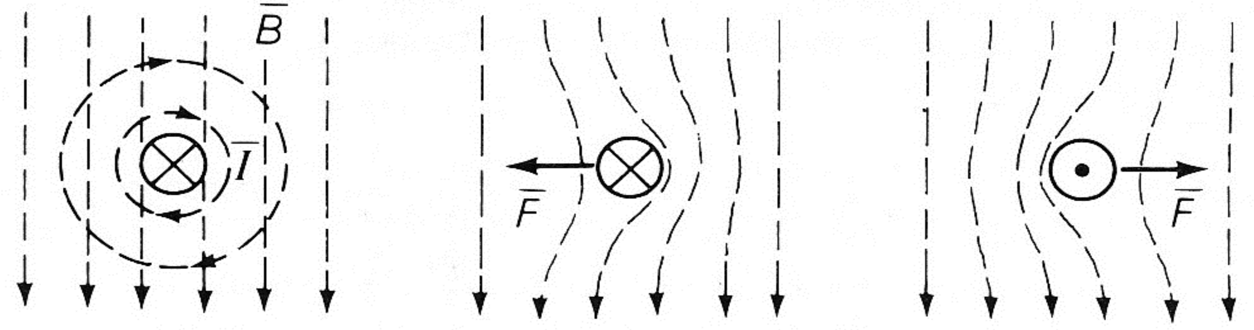
(b)

The change of flux linkage in a coil may occur in one of the following three ways:

a. The flux remaining constant, the coil moves through it.
b. The coil remaining stationary with respect to the flux, the flux varies in magnitude with time.
c. The coil may move through a time-varying flux; that is to say, both changes may occur together.

Principle of interaction

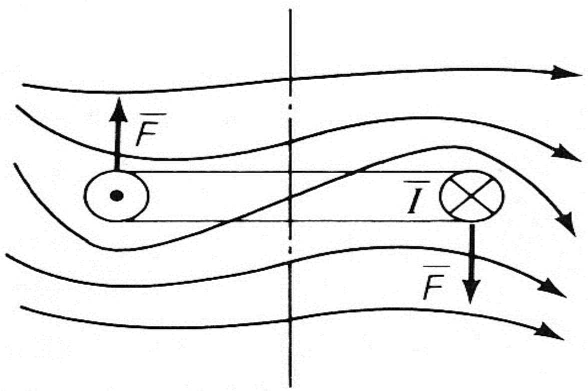
- **PRINCIPLE OF INTERACTION**



(a) (b) (c)

Consider the flux density $\overline{B}$ of an undisturbed uniform field, shown in Figure (*a*), on which the introduction of a current-carrying conductor imposes a corresponding field component, developing the resultant as in Figure (*b*) for the case in which the current is directed into and perpendicular to the plane of the paper, as symbolized by the cross in the figure. In the neighborhood of the conductor, as seen in Figure (*b*), the resultant flux density is greater than B on one side and less than B on the other side. Figure (*c*) shows the conditions corresponding to the current's being directed out of and perpendicular to the plane of the paper, as symbolized by the dot. The direction of the mechanical force developed is such that it tends to restore the field to its original undisturbed and uniform configuration, as shown in Figures (*b*) and (*c*).

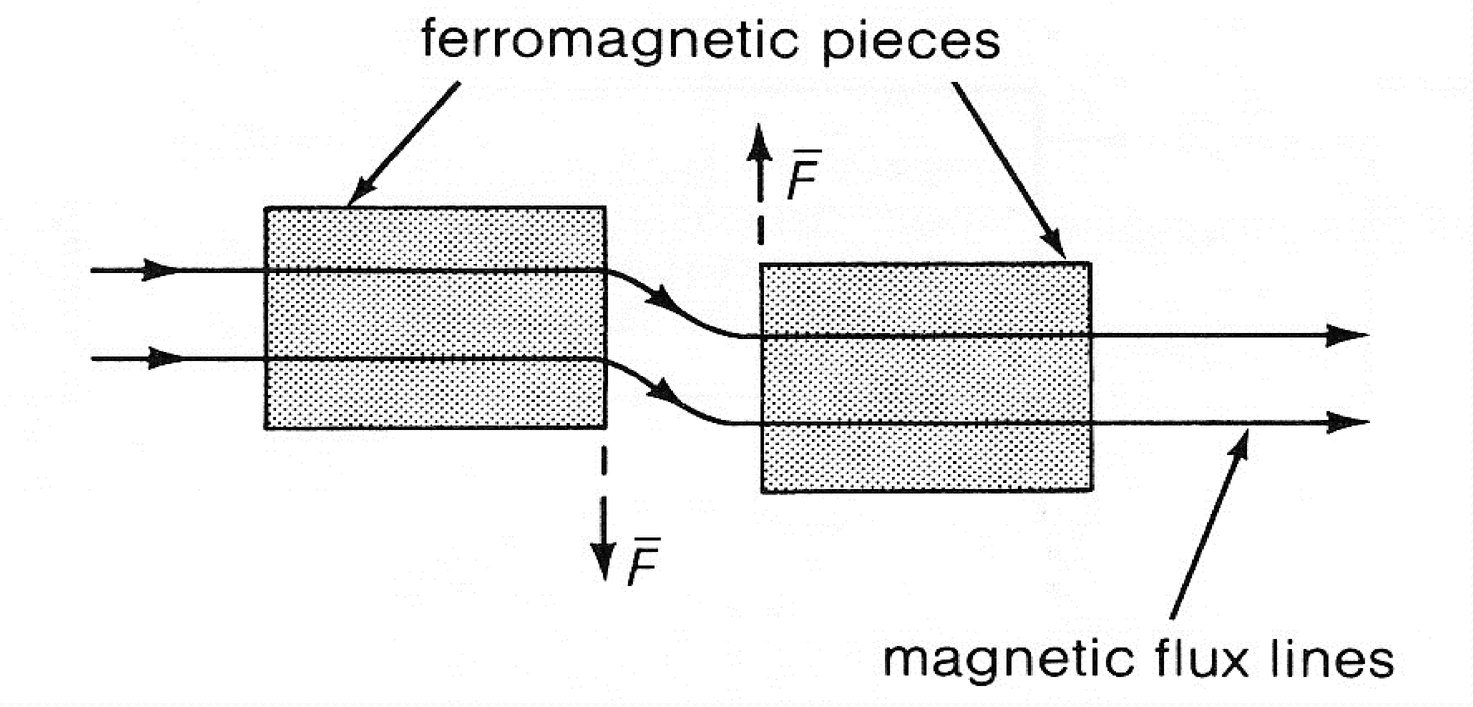
Torque produced by forces caused by interaction of current-carrying conductors and magnetic fields.



The figure indicates a one-turn coil in a magnetic field, and illustrates how torque is produced by forces caused by interaction of current-carrying conductors and magnetic fields.

Principle of alignment

PRINCIPLE OF ALIGNMENT



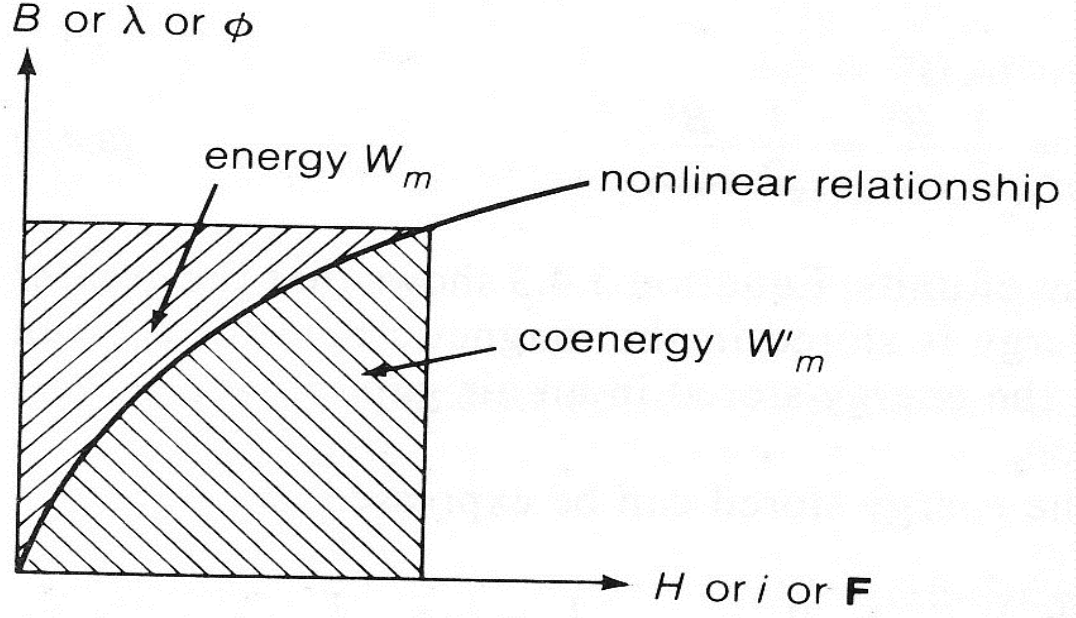
Pieces of highly permeable material such as iron situated in an ambient medium of low permeability, such as air, in which a magnetic field is established, experience mechanical forces that tend to align them with the field direction in such a way that the reluctance of the system is minimized. This principle of alignment is illustrated in Figure , showing the direction of forces. The force is always in such a direction that the net magnetic reluctance is reduced, and the magnetic flux path is shortened.

With this general background we shall proceed to evaluate the electromagnetic forces and torques associated with magnetic-field systems.

Energy stored in magnetic field

ENERGY STORED IN MAGNETIC FIELD

Graphical interpretation of energy and coenergy in a singly excited nonlinear system.



- *Since* $v = R\,i + e$
- $v\,i = R\,i^2 + e\,i$
- Where ($v\,i$) is the electric input, ($R\,i^2$)is the losses in R, and consequently ($e\,i$) is the power going to magnetic circuit.
- The energy is given by:
- Energy = $\int power\ dt$
- If the current increases from 0 to i from t=0 to t, then

$$W_m = \int_0^t e\,i\,dt = \int_0^t \frac{d\lambda}{dt}\,i\,dt = \int_0^\lambda i\,d\lambda = \int_0^\lambda i\,d(N\phi) = \int_0^\phi Ni\,d\phi = \int_0^\phi F\,d\phi$$

$$= \int_0^\Phi H\,l\,d(BA) = A\,l\int_0^B H\,dB = Volume\int_0^B H\,dB$$

For air $\mu_r = 1$

$$\therefore B = \mu_o H$$

$$W_m = Volume\int_0^B \frac{B}{\mu_o}\,dB = \frac{1}{2}\,\frac{B^2}{\mu_o} \times Volume = \frac{1}{2}\,\mu_o H^2 \times Volume = \frac{1}{2}\,B\,H \times Volume$$

$$= \frac{1}{2}\,B\,H \times A\,l = \frac{1}{2}\,F\,\phi = \frac{1}{2}\,N\,i\,\phi = \frac{1}{2}\,\lambda\,i$$

Coenergy W_m'

- The coenergy is defined as:

$$W_m' = \lambda\, i - W_m = \int_0^i \lambda\, di = \int_0^F \phi\, dF = Volume \int_0^H B\, dH$$

$$\&\quad W_m + W_m' = \lambda\, i$$

For a linear magnetic system, the λ-i characteristic is a straight line, in which case the magnetic energy and coenergy are always equal in magnitude.

$$W_m = W_m' = \frac{1}{2}\lambda\, i = \frac{1}{2} F\,\phi = \frac{1}{2} B\, H \times Volume = \frac{1}{2} L\, i^2$$

Forces and torques in magnetic field systems

FORCES AND TORQUES IN MAGNETIC FIELD SYSTEMS

For the case of a *sink* of electrical energy, such as an electric motor, the principle of conservation of energy allows one to write:

Electrical input energy from source	=	Mechanical output energy to load	+	Increase in stored field energy	+	Energy loss converted to heat

The energy losses associated with this form of energy conversion are *(a)* the energy loss due to the resistances of the circuits, *(b)* the energy loss due to friction and windage associated with motion, and *(c)* the energy loss associated with the coupling field. Considering the coupling field to be a magnetic field, the field losses are due to hysteresis and eddy-current losses, i.e., the core losses in the magnetic system. Since these losses are usually small, they may be neglected, or their effect may be included in the lossy portion of the electrical system. Then, considering only the conservative (or *lossless*) portion of the system, one has

Electrical energy from source minus electrical system losses	=	Mechanical energy to load plus mechanical system losses	+	Increase in magnetic-coupling field energy stored

or

Input electrical energy to the lossless electromechanical system	=	Mechanical work done	+	Increase in stored energy

The increase in energy stored in the magnetic field is considered here, while neglecting the energy stored in the electric field. In incremental form, in time dt,

$$dW_e = dW + dW_m$$

$$v\,i\,dt = -F\,dx + dW_m$$

where $(-F\,dx)$ corresponds to the mechanical output of the lossless electromechanical system, which may also be expressed as $(F_e\,dx)$, in which F_e is the mechanical force of electrical origin due to magnetic field coupling. Then it follows

$$F_e\,dx = v\,i\,dt - dW_m$$

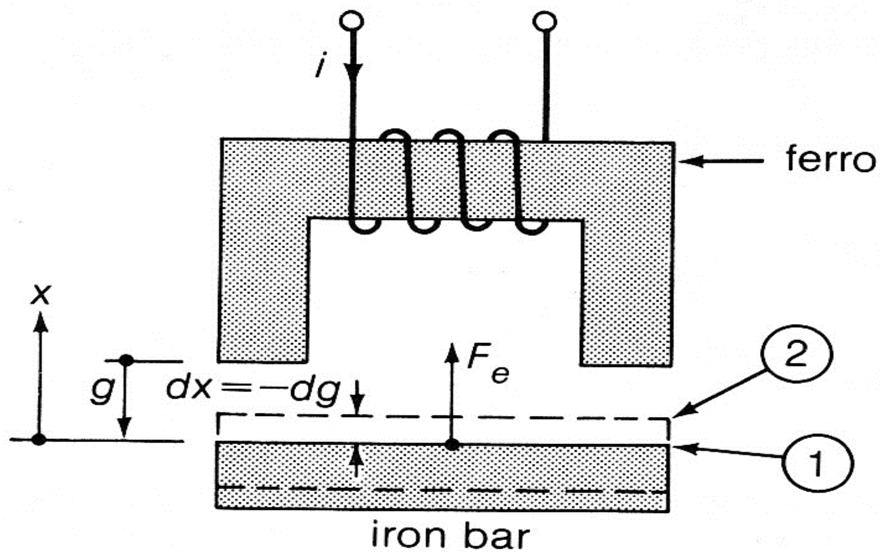
where dW_m is the increase in energy stored in the magnetic field, and dx is an arbitrary incremental displacement. Since v, which is the same as the induced emf in the lossless electromechanical system, can be expressed in terms of the flux linkage λ by means of Faraday's law of induction as

$$v = \frac{d\lambda}{dt}$$

$$F_e\,dx = i\,d\lambda - dW_m$$

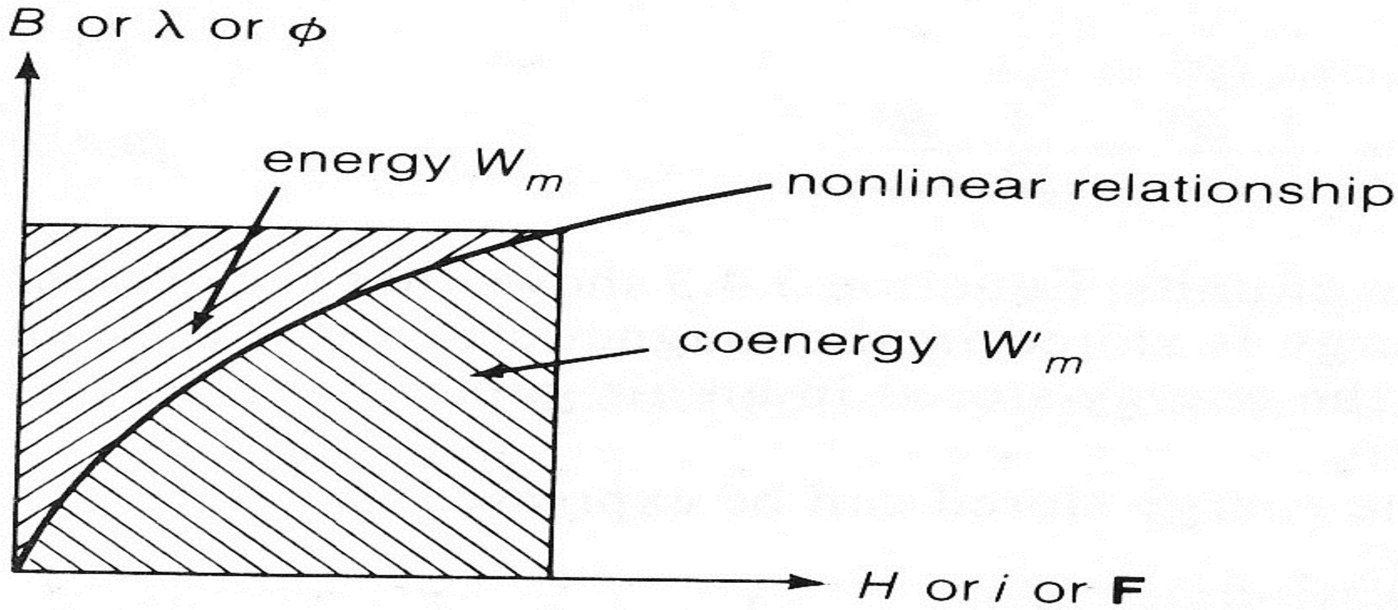
In a simple electromechanical system with a singly-excited electrical system and a mechanical system consisting of only one-dimensional motion, such as the one shown in Figure , the independent variable on the electrical side is either the current i or the flux linkage λ.

A simple electromechanical system.



The λ-i characteristic of the nonlinear magnetic system with no core loss is given by a single-valued nonlinear relationship, typically shown in Figure.

Graphical interpretation of energy and coenergy in a singly excited nonlinear system.



- *Thus, if the independent variables are the current i and the coordinate x, then the flux linkage λ is a function of both i and x:*
- $$\lambda = \lambda(i,x)$$

in which case $d\lambda$ can be expressed as

$$d\lambda = \frac{\partial \lambda}{\partial i} di + \frac{\partial \lambda}{\partial x} dx$$

where di is an arbitrary incremental change in i. Also, since the stored magnetic energy is also a function of i and x, it follows that

$$dW_m = \frac{\partial W_m}{\partial i} di + \frac{\partial W_m}{\partial x} dx$$

Substituting, we get:

$$F_e\, dx = \left(-\frac{\partial W_m}{\partial x} + i\,\frac{\partial \lambda}{\partial x}\right) dx + \left(-\frac{\partial W_m}{\partial i} + i\,\frac{\partial \lambda}{\partial i}\right) di$$

In order that the force F_e be independent of the change in the current i (and λ) during the arbitrary displacement, the coefficient of di must be zero. Consequently, the force F_e is always given by

$$F_e = -\frac{\partial W_m(i, x)}{\partial x} + i\,\frac{\partial \lambda(i, x)}{\partial x}$$

in which W_m and λ are functions of independent variables i and x. It is possible to express Equation in a simpler form in terms of magnetic coenergy W_m'

$$F_e = +\frac{\partial W_m'(i, x)}{\partial x}$$

If, on the other hand, the independent variables are chosen as λ and x, it follows that

$$i = i(\lambda, x)$$

$$di = \frac{\partial i}{\partial \lambda} d\lambda + \frac{\partial i}{\partial x} dx$$

$$dW_m = \frac{\partial W_m}{\partial \lambda} d\lambda + \frac{\partial W_m}{\partial x} dx$$

$$F_e\, dx = i\, d\lambda - \frac{\partial W_m}{\partial \lambda} d\lambda - \frac{\partial W_m}{\partial x} dx$$

or

$$F_e = -\frac{\partial W_m(\lambda, x)}{\partial x}$$

Note that W_m in this equation is a function of independent variables λ and x. This equation may also be written as:

$$F_e = \frac{\partial W_m'(\lambda, x)}{\partial x} - \lambda \frac{\partial i(\lambda, x)}{\partial x}$$

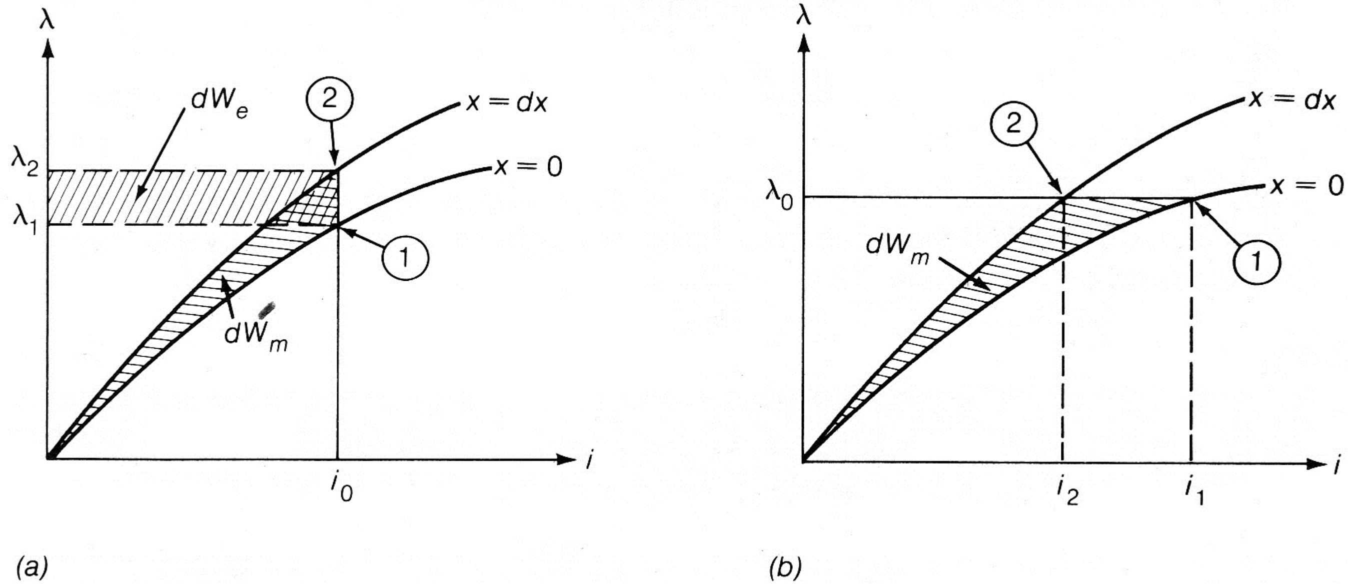
MECHANICAL FORCE OF ELECTRICAL ORIGIN CAUSED BY THE MAGNETIC COUPLING FIELD

Stored magnetic energy	$W_m = \int_0^\lambda i \, d\lambda$
Magnetic coenergy	$W_m' = \int_0^i \lambda \, di$
Relation between energy and coenergy	$W_m + W_m' = \lambda i$
Conservation of energy principle applied to conservative coupling fields for an arbitrary displacement dx	$F_e \, dx = i \, d\lambda - dW_m$

Independent Variables	Electromechanical Coupling Force Evaluated from Stored Magnetic Energy	Electromechanical Coupling Force Evaluated from Magnetic Coenergy
Current i, Coordinate x	$F_e = -\dfrac{\partial W_m(i,x)}{\partial x} + i\dfrac{\partial \lambda(i,x)}{\partial x}$	$F_e = +\dfrac{\partial W_m'(i,x)}{\partial x}$
Flux Linkage λ, Coordinate x	$F_e = -\dfrac{\partial W_m(\lambda,x)}{\partial x}$	$F_e = \dfrac{\partial W_m'(\lambda,x)}{\partial x} - \lambda\dfrac{\partial i(\lambda,x)}{\partial x}$

Note: For the case of a rotational electromechanical system, the force F_e and the linear displacement dx are to be replaced by the torque T_e and the angular displacement $d\theta$, respectively.

Energy balance in a nonlinear electromechanical system:
(a) constant current operation, *(b)* constant flux linkage (or voltage) operation



For a linear magnetic system, the λ-i characteristic is a straight line, in which case the magnetic energy and coenergy are always equal in magnitude.

$$\therefore W_m = W'_m = \frac{1}{2}\lambda i = \frac{1}{2} F \phi = \frac{1}{2} B H \times Volume = \frac{1}{2} L i^2$$

$$W_m = \frac{1}{2} i\lambda = \frac{Li^2}{2}$$

from which

$$F_e = \frac{1}{2} i^2 \frac{\partial L}{\partial x} = -\frac{1}{2}\lambda \frac{\partial i}{\partial x}$$

It should now be clear that, in order for energy conversion to take place, the electromechanical device must have at least one component capable of storing energy.

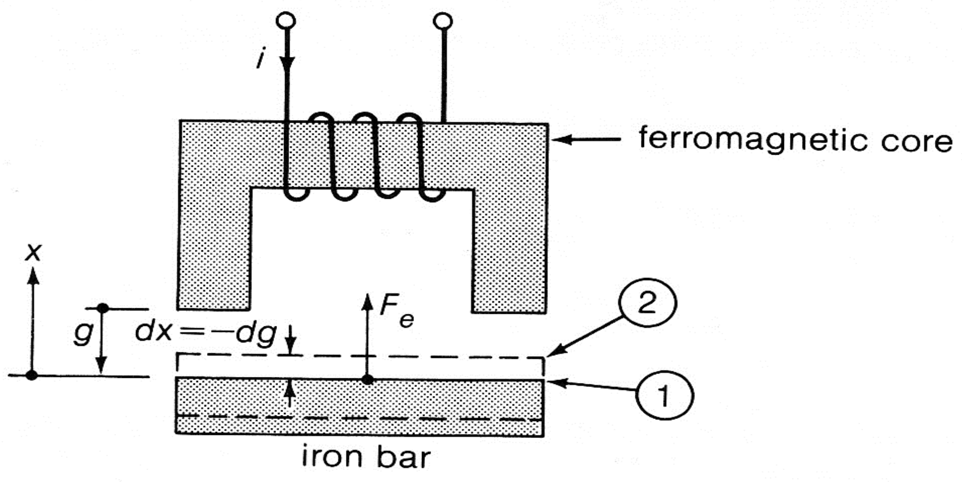
While the foregoing analysis is concerned with the force F_e and the linear displacement dx, for the case of a rotational electromechanical system, the torque T_e and the angular displacement $d\theta$ must be introduced.

Example (1)

Example (1)

The λ-i relationship for an electromechanical system show is given by

A simple electromechanical system.



$$\lambda = \frac{0.1 i^{1/2}}{g}$$

which holds good between the limits $0 < i < 4$ A and $0.04 < g < 0.10$ m. If the current is maintained at 2A, find the mechanical force (of electrical origin caused by the magnetic coupling field) on the iron bar for $g = 0.06$ m.

Solution

$$W_m' = \int_0^i \lambda \, di = \int_0^i \frac{0.1 i^{1/2}}{g} di = \frac{0.1}{g} \frac{2}{3} i^{3/2} \text{ Joules}$$

$$F_e = \frac{\partial W_m'(i, g)}{\partial g} = \frac{\partial}{\partial g}\left[\frac{0.1}{g} \frac{2}{3} i^{3/2}\right] = -\frac{0.1}{g^2} \frac{2}{3} i^{3/2}$$

For $g = 0.06$ m and $i = 2$ A, one obtains

$$F_e = -\frac{0.1}{0.06^2} \times \frac{2}{3} \times (2)^{3/2} = -52.37 \text{ newtons}$$

The negative sign indicates that the force F_e acts in such a direction as to decrease the air-gap length g. Note that a positive displacement dx in Figure corresponds to a reduction dg in the air-gap length, i.e., $dx = -dg$.

This problem may alternatively be solved by expressing i as a function of λ and g, evaluating the magnetic energy W_m,

$$i = \left(\frac{\lambda g}{0.1}\right)^2$$

$$W_m = \int_0^\lambda i \, d\lambda = \int_0^\lambda \frac{g^2}{0.1^2} \lambda^2 \, d\lambda = \frac{g^2}{0.1^2} \frac{\lambda^3}{3}$$

$$F_e = -\frac{\partial W_m(\lambda, g)}{\partial g} = -\frac{\partial}{\partial g}\left(\frac{g^2}{0.1^2} \frac{\lambda^3}{3}\right) = -\frac{\lambda^3}{3} \cdot \frac{2g}{0.1^2}$$

When $i = 2$ A and $g = 0.06$ m,

$$\lambda = \frac{0.1 \times 2^{1/2}}{0.06}$$

and

$$F_e = -\frac{0.1^3 \times 2^{3/2}}{0.06^3} \times \frac{1}{3} \times \frac{2 \times 0.06}{0.1^2}$$

$$= -\frac{0.1}{0.06^2} \times \frac{2}{3} \times 2^{3/2} = -52.37 \text{ N}$$

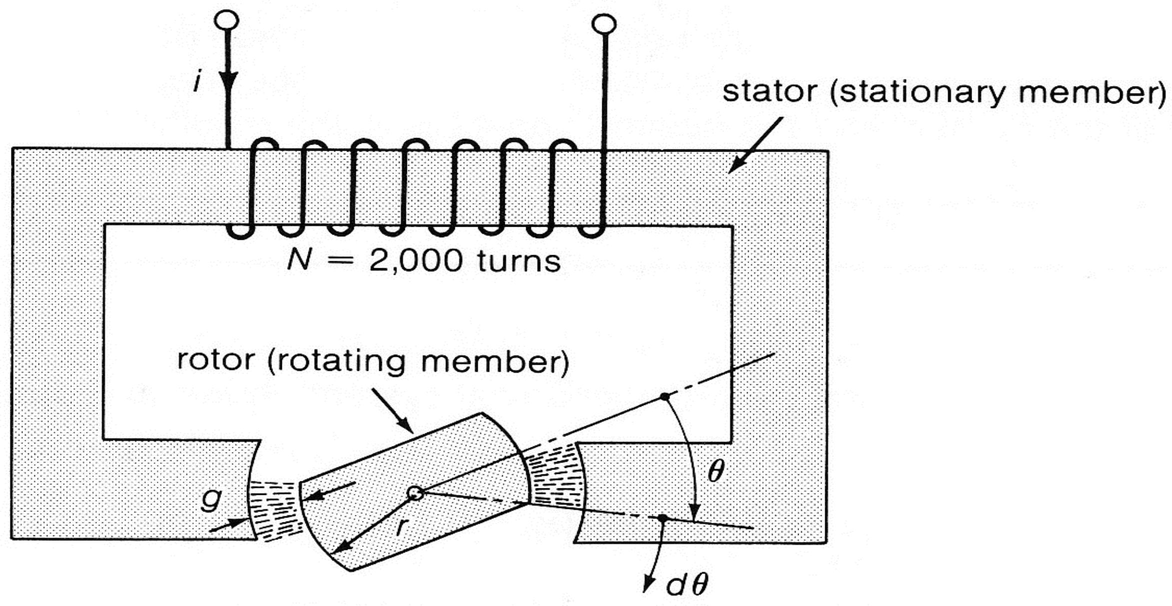
which is the same as the result obtained before. The selection of the energy or coenergy function as a basis for analysis is a matter of convenience, depending upon the initial description of the system and the desired variables in the result.

Example (2)

Example (2)

The magnetic structure shown with dimensions in the following figure is made out of a ferromagnetic material that has negligible reluctance. The rotor is free to rotate about a vertical axis. Neglect leakage and fringing.

a. Obtain an expression for the torque acting on the rotor.
b. Calculate the torque for a current of 1.5 amps and the dimensions given with the figure.
c. If the maximum flux density in the airgap is to be limited to 1.5 Wb/m^2 because of saturation in the ferromagnetic structure, compute the maximum torque of the device.



Axial length perpendicular to the plane of the paper $= h = 0.05$ m
Length of a single airgap $= g = 0.004$ m
Radius of rotor face $= r = 0.04$ m

Solution

a. Choosing as independent variables the current i and the space coordinate θ, the expression for the torque is given by

$$T_e = \frac{\partial W_m'(i, \theta)}{\partial \theta}$$

Since the air-gap region is linear, the magnetic energy or coenergy density in the air gap is given by

$$w_m = w_m' = \frac{B_g^2}{2\mu_0} = \frac{\mu_0 H_g^2}{2} \text{ joules/m}^3$$

where B_g is the air-gap flux density, H_g is the air-gap field intensity, and μ_0 is the permeability of free space. Noting that the total length of the air gap is $2g$, the volume of the overall air-gap region is calculated as

$$2gh(r + 0.5g)\,\theta \text{ m}^3$$

where θ is the angle in radians between the stator-pole tip and the adjacent rotor-pole tip as shown in the figure, and $[(r + 0.5g)\,\theta]$ is the mean arc length in the air gap. Then

$$W_m' = \mu_0 H_g^2\, gh(r + 0.5g)\,\theta \text{ joules}$$

$$\begin{aligned} T_e &= \mu_0 H_g^2\, gh(r + 0.5g) \\ &= \frac{B_g^2}{\mu_0}\, gh(r + 0.5g) \text{ newton - meters} \end{aligned}$$

The torque acts in such a direction as to align the rotor pole faces with the stator pole faces, in the positive direction of θ as shown. The relationship between the current and the air-gap field intensity H_g is given by

$$Ni = 2g\,H_g$$

or

$$H_g = \frac{Ni}{2g}$$

Making use of the above, the mechanical torque T_e of electrical origin caused by the magnetic coupling field may be expressed as

$$T_e = \frac{\mu_0 N^2 i^2}{4g^2} gh(r + 0.5g)$$

or

$$T_e = \frac{i^2}{2}\left[\frac{\mu_0 N^2}{2g} h(r + 0.5g)\right]$$

For a linear magnetic system, T_e may also be calculated from

$$T_e = \frac{i^2}{2}\frac{dL}{d\theta}$$

For our problem,

$$L = \frac{N^2}{\mathbf{R}} = \frac{\mu_0 A N^2}{l}$$

where

$$\begin{aligned} A &= \text{cross-sectional area of the air-gap region} \\ &= h(r + 0.5\, g)\, \theta, \end{aligned}$$

and

$$l = 2\, g, \text{ the total air-gap length}$$

The same result for T_e is obtained by substituting and working out the details.

b. The self-inductance L in terms of θ is given by

$$L = \frac{4\pi \times 10^{-7} \times 0.05(0.04 + 0.002) \times 2{,}000^2}{0.008}\,\theta \text{ henry}$$

or

$$L = 1.32\theta$$

$$T_e = \frac{i^2}{2} \cdot \frac{dL}{d\theta} = \frac{(1.5)^2}{2} \times 1.32 = 1.485 \text{ newton-meters}$$

Note that the torque acts to increase the inductance by pulling on the rotor so as to reduce the reluctance of the magnetic path linking the coil.

The air-gap flux density B_g corresponding to a current of 1.5 amps is given by

$$B_g = \mu_0 H_g = \frac{\mu_0 Ni}{2g} = \frac{4\pi \times 10^{-7} \times 2{,}000 \times 1.5}{2 \times 0.004} = 0.47 \text{ T}$$

c. Corresponding to $B_{g_{max}}$ of 1.5 Wb/m^2, the current is calculated as

$$i_{max} = \frac{B_g(2g)}{\mu_0 N} = \frac{1.5 \times 0.008}{4\pi \times 10^{-7} \times 2{,}000} = 4.77 \text{ A}$$

$$T_{e_{max}} = \frac{i^2}{2}\frac{dL}{d\theta} = \frac{4.77^2}{2} \times 1.32 = 15.02 \text{ N}\cdot\text{m}$$

This may also be obtained by directly substituting in the torque expression given in terms of B_g.

Singly excited magnetic field systems

Before we proceed to analyze magnetic field systems excited by more than one electrical circuit, let us consider an elementary reluctance machine that is singly excited, carrying only one winding on its stationary member, called the stator.

Figure (a), shows an elementary rotating reluctance machine. We shall assume that the reluctances of the stator and rotor iron are negligible; also, we shall neglect the leakage and fringing.

The stator and rotor poles are so shaped that the reluctance varies sinusoidally about a mean value as shown in Figure (b). $\mathbf{R}_d$ is the reluctance of the magnetic system when the rotor is in the direct-axis position ($\theta = 0$), and $\mathbf{R}_q$ is the reluctance when the rotor is in the quadrature-axis position ($\theta = \pi/2$). For each revolution of the rotor, there are two cycles of reluctance. The space variation of inductance is also of double frequency, since the inductance is inversely proportional to the reluctance. The inductance of the stator winding as a function of space coordinate θ measured from the direct axis, as shown in Figure (a) is given by

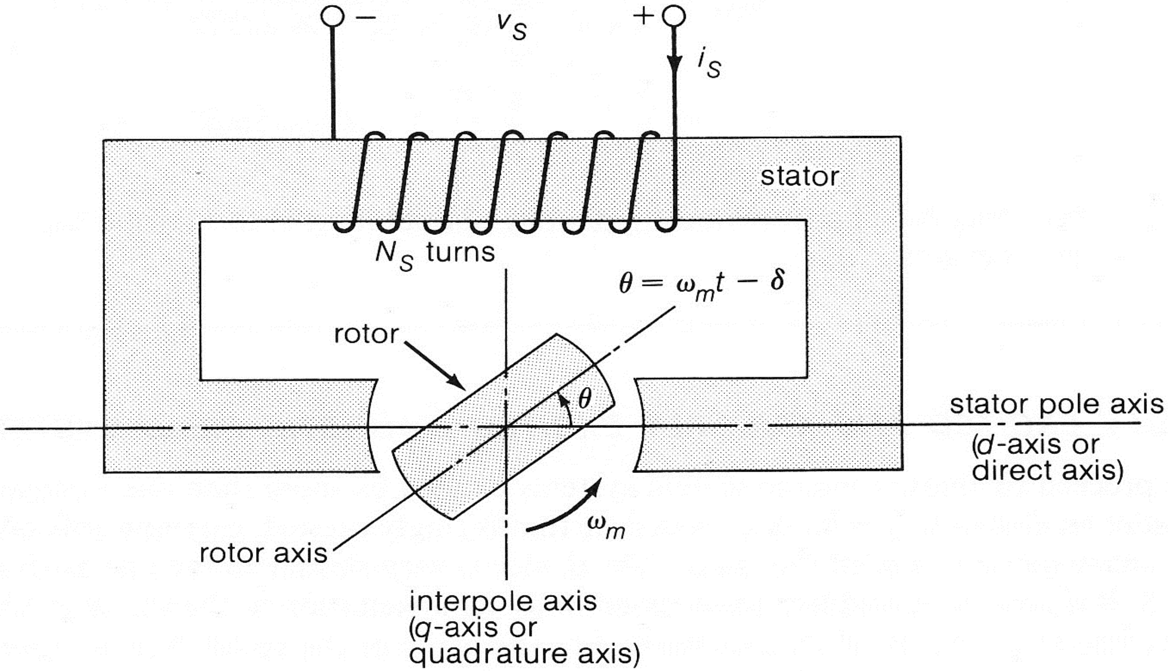
$$L(\theta) = L_0 + L_2 \cos 2\theta$$

which is sketched in Figure (c). Let the stator coil excitation be

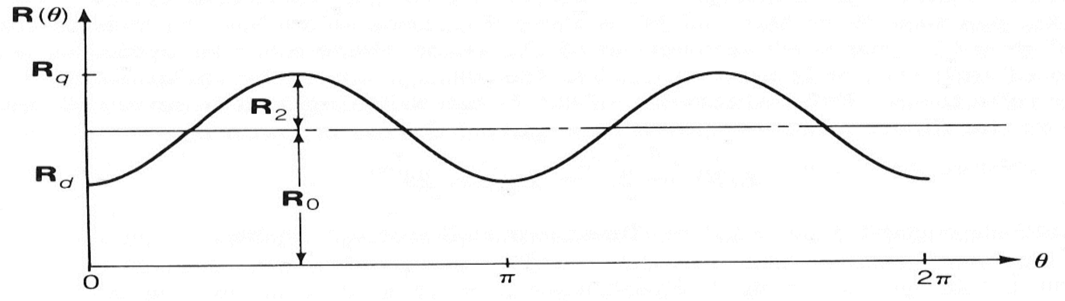
$$i_s = I_s \sin \omega_s t$$

We shall investigate the instantaneous and average electromagnetic torques produced because of this sinusoidal excitation whose angular frequency is ω_s.

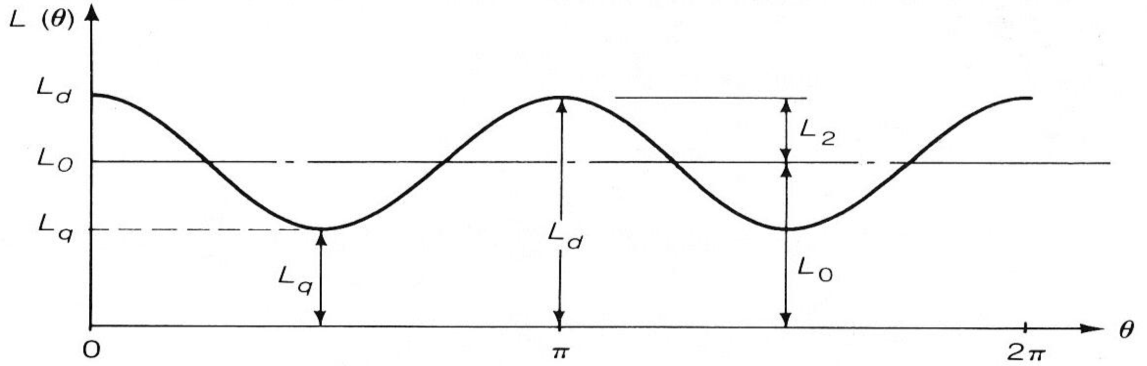
A singly-excited magnetic field system with a rotor: *(a)* an elementary rotating reluctance machine, *(b)* reluctance variation with rotor position, *(c)* inductance variation with rotor position.



(a)



(b)



(c)

The electromagnetic torque can be found from the coenergy in the magnetic field of the air-gap region, since the independent variables are the current i and the space coordinate θ.

$$W_m' = W_m = \frac{1}{2} L(\theta)\, i_s^2$$

and

$$T_e = \frac{\partial W_m'(i_s, \theta)}{\partial \theta} = \frac{1}{2} i_s^2 \frac{\partial L(\theta)}{\partial \theta}$$

Substituting the current and inductance variations, one obtains

$$T_e = -I_s^2 L_2 \sin 2\theta \sin^2 \omega_s t$$

Let the rotor now be allowed to rotate at an angular velocity ω_m, so that at any instant θ is given by

$$\theta = \omega_m t - \delta$$

where $\theta = -\delta$ is the angular position of the rotor at $t = 0$, when the current i_s is zero.

$$\sin^2 A = \frac{1 - \cos 2A}{2}$$

and

$$\sin A \cos B = \frac{1}{2} \sin(A + B) + \frac{1}{2} \sin (A - B)$$

The instantaneous electromagnetic torque is then given by

$$\begin{aligned} T_e = -\frac{1}{2} I_s^2 L_2 \Big\{ & \sin 2(\omega_m t - \delta) \\ & - \frac{1}{2} \sin 2[(\omega_m + \omega_s) t - \delta] \\ & - \frac{1}{2} \sin 2[(\omega_m - \omega_s) t - \delta] \Big\} \end{aligned}$$

The above torque expression consists of three sinusoidally time-varying terms of frequencies $2\omega_m$, $2(\omega_m + \omega_s)$ and $2(\omega_m - \omega_s)$. The time-average value of these three sine terms is zero unless, in one of them, the coefficient of t becomes zero. Since $\omega_m \neq 0$, the necessary condition for nonzero time-average torque is then

$$|\omega_m| = |\omega_s|$$

corresponding to which

$$(T_e)_{av} = -\frac{1}{4} I_s^2 L_2 \sin 2\delta$$

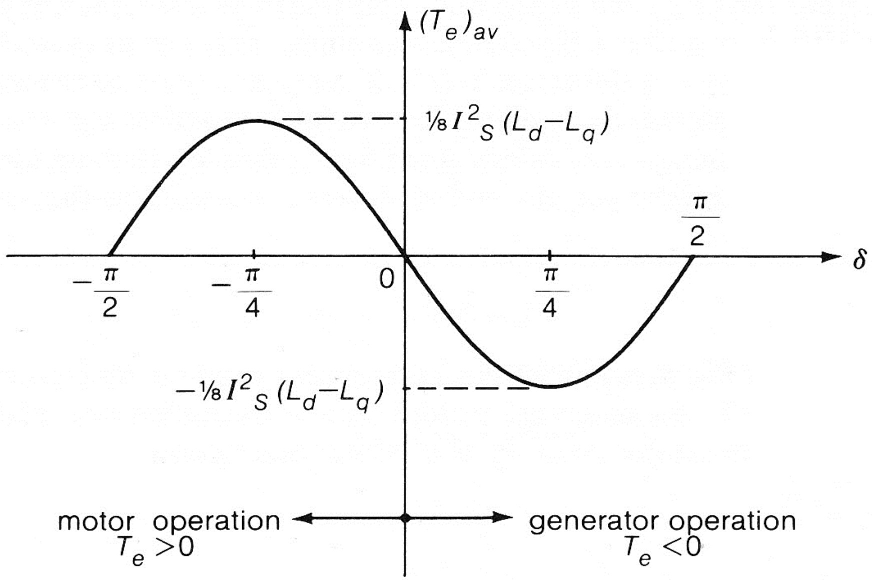
Expressed in terms of L_d and L_q, the maximum and minimum values of inductance as shown in Figure (c), known as the direct-axis inductance and quadrature-axis inductance, respectively,

$$(T_e)_{av} = -\frac{1}{8} I_s^2(L_d - L_q) \sin 2\delta$$

Since the torque in this particular electromechanical energy converter is due to the variation of reluctance with rotor position, the device is known as a synchronous reluctance machine. As seen from equation, the torque is zero if $L_d = L_q$, i.e. if there is no inductance or reluctance variation with rotor position. The Figure shows the variation of the average electromechanical torque developed by the machine as a function of the angle δ, which is known as the torque angle.

It should be emphasized that the singly excited synchronous reluctance motor cannot start by itself.

Variation of the electromagnetic torque developed by a synchronous reluctance machine.



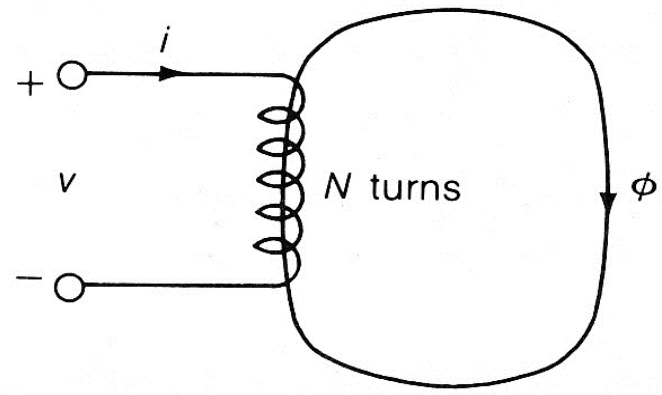
Inductance

INDUCTANCE

An electric circuit in which the current links the magnetic flux is said to have *inductance*. A single loop or an inductor carrying a current i is linked by its own flux, as shown in Figure . If the medium in the flux path has a linear magnetic characteristic (i.e., constant permeability), then the relationship between the flux linkages λ and the current i is linear, and the slope of the linear λ-i characteristic gives the *self-inductance,* defined as flux linkage per ampere:

$$L = \frac{\lambda}{i} = \frac{N\phi}{i} = \frac{N^2}{\mathbf{R}}$$

A single inductive loop.

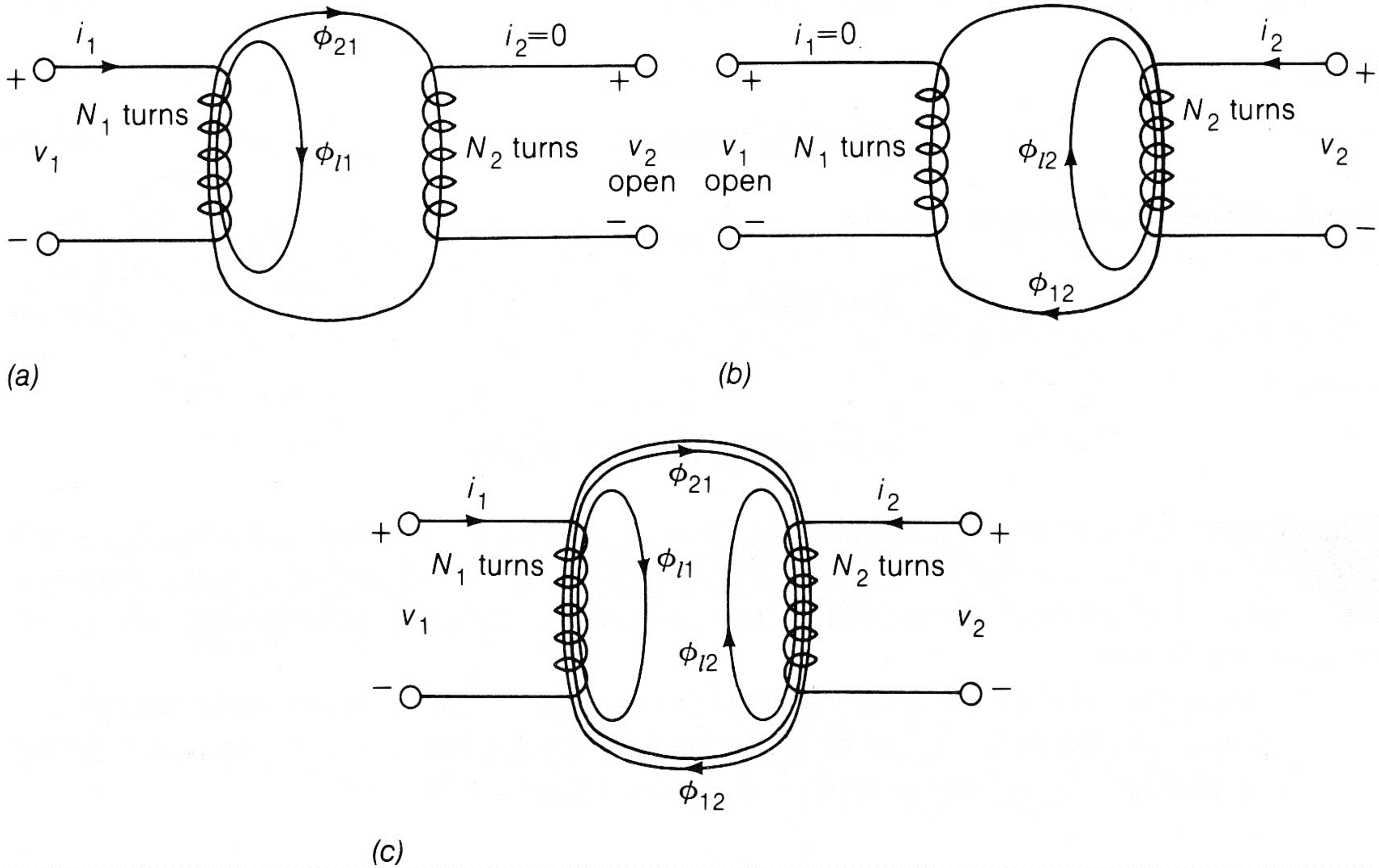


The above equation illustrates that the inductance is a function of the geometry and permeability, and that in a linear system, it is independent of voltage, current, and frequency. The energy stored in the magnetic field of an inductor in a linear medium is given by

$$W_m = \frac{1}{2} i\lambda = \frac{Li^2}{2}$$

When more than one loop or circuit is present, the flux produced by the current in one loop may link another loop, thereby inducing a current in that loop; such loops are said to be mutually coupled, and there exists a *mutual inductance* between such loops. The mutual inductance between two circuits is defined as the flux linkage produced in one circuit by a current of one ampere in the other circuit. Let us now consider a pair of mutually coupled inductors, as shown in Figure

Mutually coupled inductors.



The self-inductances L_{11} and L_{22} of inductors 1 and 2 respectively are given by

$$L_{11} = \frac{\lambda_{11}}{i_1}$$

and

$$L_{22} = \frac{\lambda_{22}}{i_2}$$

where λ_{11} is the flux linkage of inductor 1 produced by its own current i_1, and λ_{22} is the flux linkage of inductor 2 produced by its own current i_2. The mutual inductances L_{12} and L_{21} are given by

$$L_{12} = \frac{\lambda_{12}}{i_2}$$

and

$$L_{21} = \frac{\lambda_{21}}{i_1}$$

where λ_{12} is the flux linkage of inductor 1 produced by the current i_2 in inductor 2, and λ_{21} is the flux linkage of inductor 2 produced by the current i_1 in inductor 1.

If a current of i_1 flows in inductor 1 while the current in inductor 2 is zero, the equivalent fluxes are given by

$$\phi_{11} = \frac{\lambda_{11}}{N_1}$$

and

$$\phi_{21} = \frac{\lambda_{21}}{N_2}$$

where N_1 and N_2 are the number of turns of inductors 1 and 2 respectively. That part of the flux of inductor 1 that does not link any turn of inductor 2 is known as the equivalent *leakage flux* of inductor 1:

$$\phi_{l1} = \phi_{11} - \phi_{21}$$

Similarly,

$$\phi_{l2} = \phi_{22} - \phi_{12}$$

The *coefficient of coupling* is given by

$$k = \sqrt{k_1 k_2}$$

where

$$k_1 = \phi_{21}/\phi_{11} \text{ and } k_2 = \phi_{12}/\phi_{22}$$

When k approaches unity, the two inductors are said to be tightly coupled; and when k is much less than unity, they are said to be loosely coupled. While the coefficient of coupling can never exceed unity, it may be as high as 0.998 for iron-core transformers; it may be smaller than 0.5 for air-core transformers.

When there are only two inductively coupled circuits, the symbol M is frequently used to represent the mutual inductance; it can be shown that the mutual inductance between two electric circuits coupled by a homogeneous medium of constant permeability is reciprocal:

$$M = L_{12} = L_{21} = k\sqrt{L_{11} L_{22}}$$

The energy considerations that lead to such a conclusion are taken up in a problem at the end of the chapter as an exercise for the student.

Let us next consider the energy stored in a pair of mutually coupled inductors:

$$W_m = \frac{i_1 \lambda_1}{2} + \frac{i_2 \lambda_2}{2}$$

where λ_1 and λ_2 are the total flux linkages of inductors 1 and 2 respectively.

$$W_m = \frac{i_1}{2}(\lambda_{11} + \lambda_{12}) + \frac{i_2}{2}(\lambda_{22} + \lambda_{21})$$

$$= \frac{1}{2} L_{11}\, i_1^2 + \frac{1}{2} L_{12}\, i_1\, i_2 + \frac{1}{2} L_{22}\, i_2^2 + \frac{1}{2} L_{21}\, i_1\, i_2$$

or

$$W_m = \frac{1}{2} L_{11}\, i_1^2 + M i_1\, i_2 + \frac{1}{2} L_{22}\, i_2^2$$

Going back to the pair of mutually coupled inductors shown in Figure , the flux-linkage relations and the voltage equations for circuits 1 and 2 are given by the following, while neglecting the resistances associated with the coils:

$$\lambda_1 = \lambda_{11} + \lambda_{12} = L_{11}\, i_1 + L_{12}\, i_2 = L_{11}\, i_1 + M i_2$$

$$\lambda_2 = \lambda_{21} + \lambda_{22} = L_{21}\, i_1 + L_{22}\, i_2 = M i_1 + L_{22}\, i_2$$

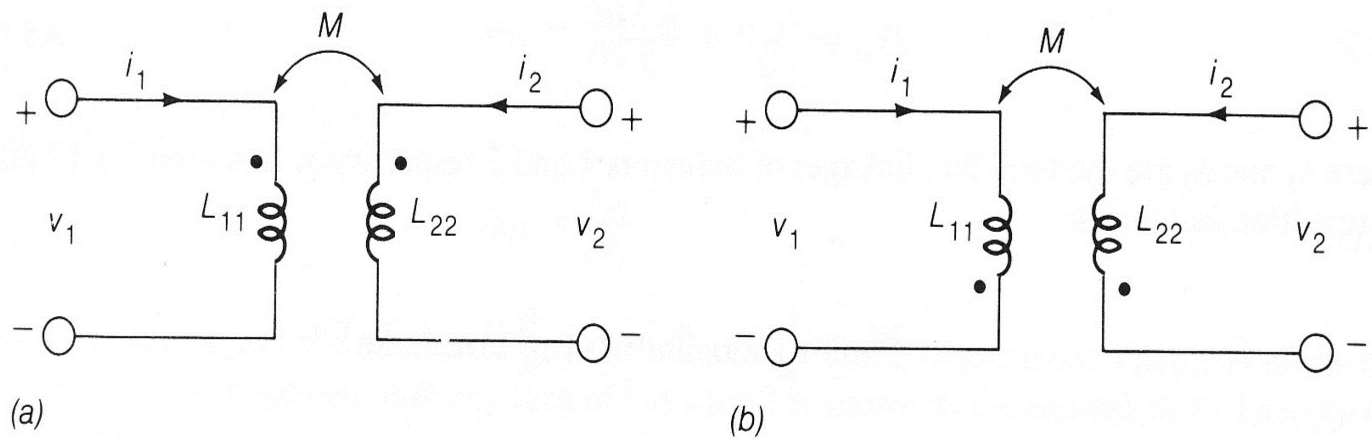
$$v_1 = p\lambda_1 = \frac{d\lambda_1}{dt} = L_{11}\frac{di_1}{dt} + M\frac{di_2}{dt}$$

$$v_2 = p\lambda_2 = \frac{d\lambda_2}{dt} = M\frac{di_1}{dt} + L_{22}\frac{di_2}{dt}$$

where p is the derivative operator d/dt.

In order to avoid drawing detailed sketches of windings showing the sense in which each coil is wound, a *dot convention* is developed.

Dot notation for a pair of mutually coupled inductors: *(a)* dots on upper terminals, *(b)* dots on lower terminals.

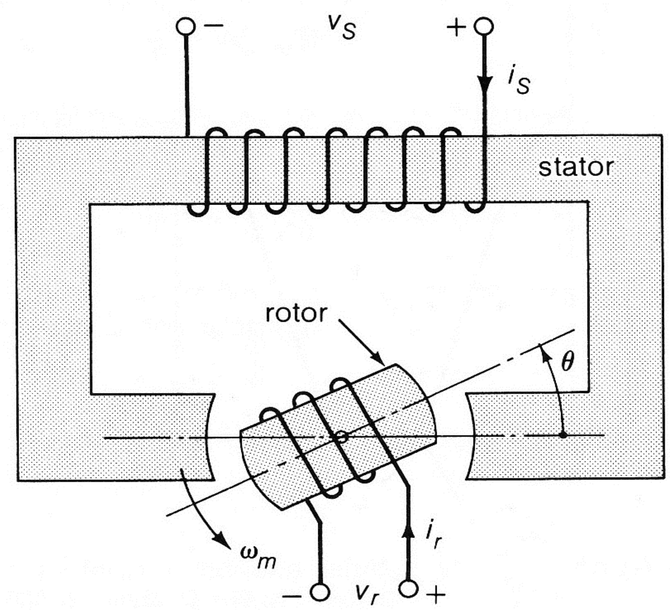


Multiply excited magnetic field systems

MULTIPLY EXCITED MAGNETIC FIELD SYSTEMS

Let us consider an elementary multiply excited magnetic system shown in Figure with two sets of electrical terminals and one mechanical terminal.

An elementary doubly-excited magnetic-field system.



The flux linkages of the stator and rotor windings can be expressed as functions of the coil currents:

$$\lambda_s = L_{ss}\, i_s + L_{sr}\, i_r$$

$$\lambda_r = L_{sr}\, i_s + L_{rr}\, i_r$$

where L_{ss} and L_{rr} are the self-inductances of the stator and rotor coils respectively, and L_{sr} is the stator-rotor mutual inductance. All these inductances are generally functions of the angle θ between the magnetic axes of the stator and rotor windings.

$$\begin{aligned} v_s &= R_s\, i_s + p\,\lambda_s \\ &= R_s\, i_s + L_{ss}(p\, i_s) + i_s(p\, L_{ss}) + L_{sr}(p\, i_r) + i_r(p\, L_{sr}) \end{aligned}$$

$$\begin{aligned} v_r &= R_r\, i_r + p\,\lambda_r \\ &= R_r\, i_r + L_{sr}(p\, i_s) + i_s(p\, L_{sr}) + L_{rr}(p\, i_r) + i_r(p\, L_{rr}) \end{aligned}$$

Neglecting the reluctances of the stator- and rotor-iron circuits, the electromagnetic torque can be found either from the energy or coenergy stored in the magnetic field of the air-gap region:

$$T_e = -\frac{\partial W_m(\lambda_s, \lambda_r, \theta)}{\partial \theta} = +\frac{\partial W_m'(i_s, i_r, \theta)}{\partial \theta}$$

For a linear system, the energy or coenergy stored in a pair of mutually coupled inductors is given by

$$W_m'(i_1, i_2, \theta) = \frac{1}{2} L_{ss}\, i_s^2 + L_{sr}\, i_s\, i_r + \frac{1}{2} L_{rr}\, i_r^2$$

The first and third terms on the right-hand side of Equation , involving angular rate of change of self-inductance are the *reluctance-torque* terms; the middle term, involving angular rate of change of mutual inductance, is the torque caused by the interaction of fields produced by the stator and rotor currents. It is this mutual-inductance torque that is most commonly exploited in practical rotating machines. Multiply excited systems with more than two sets of electrical terminals can be handled in a similar manner as for two pairs by assigning additional independent variables to the terminals.

If the self-inductances L_{ss} and L_{rr} are independent of angle θ, the reluctance torque is zero, and the torque is produced only by the mutual term $L_{sr}(\theta)$.

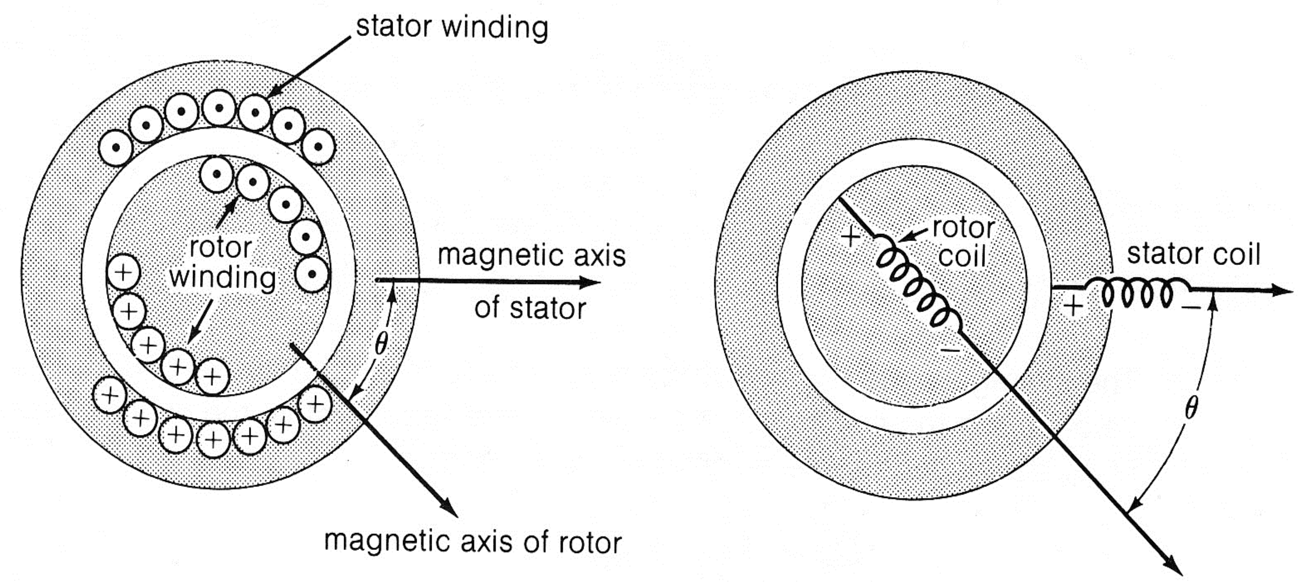
Example (3)

Example (3)

Consider an elementary two-pole rotating machine with a uniform (or smooth) air gap as shown in Figure , in which the cylindrical rotor is mounted within the stator made up of a hollow cylinder coaxial with the rotor. The stator and rotor windings are distributed over a number of slots so that their mmf waves can be approximated by space sinusoids. A consequence of such a type of construction is that one can fairly assume that the self-inductances L_{ss} and L_{rr} are constant, but the mutual inductance L_{sr} is given by

$$L_{sr} = L \cos \theta$$

An elementary two-pole rotating machine with uniform air-gap



(a)

(b)

where θ is the angle between the magnetic axes of the stator and rotor windings. Let the currents in the two windings be given by

$$i_s = I_s \cos \omega_s t$$

$$i_r = I_r \cos (\omega_r t + \alpha)$$

and let the rotor rotate at an angular velocity

$$\omega_m = \dot{\theta} \text{ rad/s}$$

such that the position of the rotor at any instant is given by

$$\theta = \omega_m t + \theta_0$$

Assume that the reluctances of the stator and rotor-iron circuits are negligible, and that the stator and rotor are concentric cylinders neglecting the effect of slot openings.

a. Derive an expression for the instantaneous electromagnetic torque developed by the machine.
b. Find the necessary condition for the development of an average torque in the machine.
c. Obtain the expression for the average torque corresponding to the following cases:

(i) $\omega_s = \omega_r = \omega_m = 0; \alpha = 0$

(ii) $\omega_s = \omega_r; \omega_m = 0$

(iii) $\omega_r = 0; \omega_s = \omega_m; \alpha = 0$

(iv) $\omega_m = \omega_s - \omega_r$, where ω_s and ω_r are two different angular frequencies

Solution

a. With constant L_{ss} and L_{rr}

$$T_e = i_s\, i_r \frac{d\, L_{sr}}{d\theta} = -i_s\, i_r\, L \sin\theta$$

when the variation of L_{sr} as a function of θ is substituted. For the given current variations, the instantaneous electromagnetic torque developed by the machine is given by

$$T_e = -L\, I_s\, I_r \cos\omega_s t \cos(\omega_r t + \alpha) \sin(\omega_m t + \theta_0)$$

Using the trigonometric identities, the product of the three trigonometric terms in the above equation may be expressed to yield

$$\begin{aligned} T_e = \frac{-L\, I_s\, I_r}{4} \Big\{ & \sin\{[\omega_m + (\omega_s + \omega_r)]t + \alpha + \theta_0\} \\ & + \sin\{[\omega_m - (\omega_s + \omega_r)]t - \alpha + \theta_0\} \\ & + \sin\{[\omega_m + (\omega_s - \omega_r)]t - \alpha + \theta_0\} \\ & + \sin\{[\omega_m - (\omega_s - \omega_r)]t + \alpha + \theta_0\} \Big\} \end{aligned}$$

b. The average value of each of the sinusoidal terms in the above equation is zero, unless the coefficient of t is zero in that term; that is, the average torque $(T_e)_{av}$ developed by the machine is zero unless

$$\omega_m = \pm(\omega_s \pm \omega_r)$$

which may also be expressed as

$$|\omega_m| = |\omega_s \pm \omega_r|$$

c. (i) The excitations are direct currents I_s and I_r. For the given conditions of $\omega_s = \omega_r = \omega_m = 0$ and $\alpha = 0$,

$$T_e = -L\, I_s\, I_r \sin\theta_0$$

which is a constant. As such

$$(T_e)_{av} = -L\, I_s\, I_r \sin\theta_0$$

The machine operates as a *dc rotary actuator,* developing a constant torque against any displacement θ_0 produced by an external torque applied to the rotor shaft.

(ii) With $\omega_s = \omega_r$, both excitations are alternating currents of the same frequency. For the conditions $\omega_s = \omega_r$ and $\omega_m = 0$

$$T_e = -\frac{L\, I_s\, I_r}{4}\left[\sin(2\omega_s t + \alpha + \theta_0) + \sin(-2\omega_s t - \alpha + \theta_0) + \sin(-\alpha + \theta_0) + \sin(\alpha + \theta_0)\right]$$

The machine operates as an *ac rotary actuator,* and the developed torque is fluctuating. The average value of the torque is

$$(T_e)_{av} = -\frac{L\, I_s\, I_r}{2} \sin\theta_0 \cos\alpha$$

Note that α becomes zero if the two windings are connected in series, in which case $\cos\alpha$ becomes unity.

(iii) With $\omega_r = 0$, the rotor excitation is a direct current I_r. For the conditions $\omega_r = 0$, $\omega_s = \omega_m$, and $\alpha = 0$,

$$T_e = -\frac{L\, I_s\, I_r}{4}[\sin(2\omega_s t + \theta_0) + \sin(\theta_0) + \sin(2\omega_s t + \theta_0) + \sin(\theta_0)]$$

or

$$T_e = -\frac{L\, I_s\, I_r}{2}[\sin(2\omega_s t + \theta_0) + \sin\theta_0]$$

The machine operates as an idealized *single-phase synchronous machine*, and the instantaneous torque is pulsating. The average value of the torque is

$$(T_e)_{av} = -\frac{L\, I_s\, I_r}{2}\sin\theta_0$$

since the average value of the double-frequency sine term is zero. If the machine is brought up to *synchronous* speed ($\omega_m = \omega_s$), an average unidirectional torque is established, and continuous energy conversion takes place at synchronous speed. Note that the machine is not self-starting, since an average unidirectional torque is not developed at $\omega_m = 0$ with the specified electrical excitations.

(iv) With $\omega_m = \omega_s - \omega_r$, the instantaneous torque is given by

$$T_e = -\frac{L I_s I_r}{4}[\sin(2\omega_s t + \alpha + \theta_0) + \sin(-2\omega_r t - \alpha + \theta_0)$$
$$+ \sin(2\omega_s t - 2\omega_r t - \alpha + \theta_0) + \sin(\alpha + \theta_0)]$$

The machine operates as a *single-phase induction machine*, and the instantaneous torque is pulsating. The average value of the torque is

$$(T_e)_{av} = -\frac{L\, I_s\, I_r}{4}\sin(\alpha + \theta_0)$$

If the machine is brought up to a speed of $\omega_m = (\omega_s - \omega_r)$, an average unidirectional torque is established, and continuous energy conversion takes place at the *asynchronous* speed of ω_m. Note that the machine is not self-starting, since an average unidirectional torque is not developed at $\omega_m = 0$ with the specified electrical excitations.

The pulsating torque, which may be acceptable in small machines, is in general an undesirable feature in a rotating machine, working either as a generator or a motor, since it may result in speed fluctuation, vibration, noise, and waste of energy. In magnetic-field systems excited by single-phase alternating sources, the torque pulsates while the speed is relatively constant; consequently, pulsating power becomes a feature. This calls for an improvement; in fact, by employing polyphase windings and polyphase sources, constant power is developed in a balanced system.